WE01 WORK, ENERGY & THE WORK-ENERGY THEOREM

SPH4U

EQUATIONS

- Work with a constant force

$$W = (F\cos\theta)\Delta d = \Delta E_K$$

- Kinetic Energy

$$E_K = \frac{1}{2}mv^2$$

WORK DONE BY A CONSTANT FORCE

- **Work (W) [J = N m]:** the energy transferred to an object when a force acting on the object moves it through a distance

$$W = (F\cos\theta)\Delta d$$

- W – work [J]
- F – magnitude of applied force $\left(\vec{F}\right)$ [N]
    - parallel to displacement
- θ – angle between force and displacement [°]
- d – magnitude of displacement $\left(\vec{d}\right)$ [m]

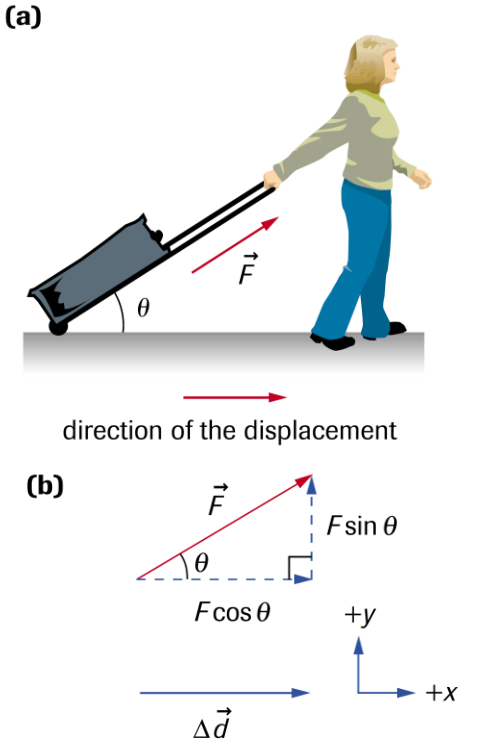


Figure 1
(a) Work can be done by a force that is at an angle to the displacement.
(b) The component of the force parallel to the displacement is $F\cos\theta$.

THE JOULE – A MEASURE OF WORK & ENERGY

- Joule [J]: SI derived unit for measuring forms of energy and work; equal to the work done when a force of 1 N displaces an object 1 m <u>in the direction of the force</u>

$$1\ \mathrm{J} = 1\ \mathrm{kg\ m^2/s^2} = 1\ \mathrm{N\ m}$$

PROBLEM 1

An emergency worker applies a force to push a patient horizontally for 2.44 m on a gurney with nearly frictionless wheels.

(a) Determine the work done in pushing the gurney if the force applied is horizontal and of magnitude of 15.5 N.

(b) Determine the work done if the force, of magnitude 15.5 N, is applied at an angle of 25.3° below the horizontal.

(c) Describe the difference in the observed motion between (a) and (b).

PROBLEM 1 – SOLUTIONS

(a) In this case, the force and the displacement are in the same direction.

$F = 15.5 \text{ N}$

$\theta = 0°$

$\Delta d = 2.44 \text{ m}$

$W = ?$

$$\begin{aligned} W &= (F \cos \theta)\Delta d \\ &= (15.5 \text{ N})(\cos 0°)(2.44 \text{ m}) \\ &= 37.8 \text{ N·m} \\ W &= 37.8 \text{ J} \end{aligned}$$

The work done is 37.8 J.

PROBLEM 1 – SOLUTIONS CONT.

(b) $\theta = 25.3°$

$W = ?$

$$W = (F \cos \theta)\Delta d$$
$$= (15.5 \text{ N})(\cos 25.3°)(2.44 \text{ m})$$
$$W = 34.2 \text{ J}$$

The work done is 34.2 J.

(c) Since friction is negligible, the applied force causes an acceleration in the direction of the horizontal component. The greater amount of work accomplished in (a) must result in a greater speed after the gurney has moved 2.44 m. (This example relates to the concept of work changing into kinetic energy, which is presented in Section 4.2.)

PROBLEM 2

Figure 5 shows a scale diagram of two applied forces, $\vec{F}_1$ and $\vec{F}_2$, acting on a crate and causing it to move horizontally. Which force does more work on the crate?

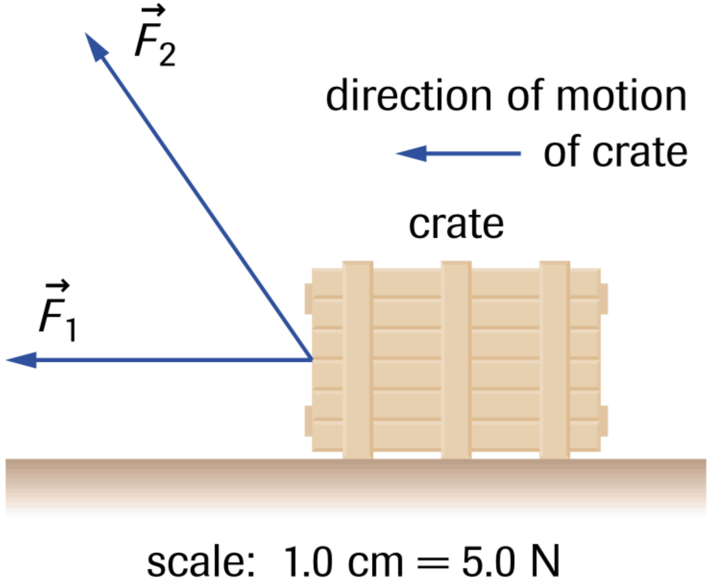


Figure 5

ZERO WORK

Conditions for zero work:

- No displacement
- Force is perpendicular to motion
  - No component of force in the direction of motion
  - $\theta = 90° \Rightarrow \cos 90° = 0$

PROBLEM 3

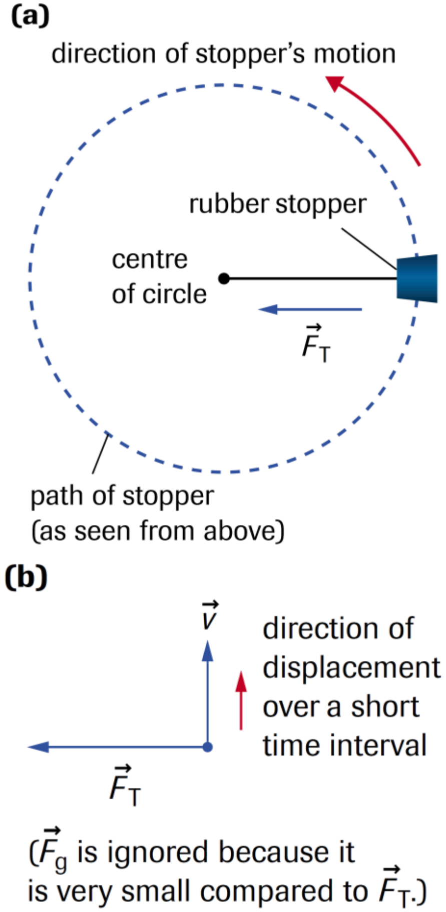
In performing a centripetal-acceleration investigation, you twirl a rubber stopper in a horizontal circle around your head. How much work is done on the stopper by the tension in the string in half a revolution?

PROBLEM 3 – SOLUTIONS

Figure 8(a) shows the situation described. The force causing the centripetal acceleration is the tension $\vec{F}_T$. This force changes direction continually as the stopper travels in a circle. However, at any particular instant as shown in **Figure 8(b)**, the instantaneous velocity is perpendicular to the tension. Consequently, the displacement over a very short time interval is also at an angle of 90° to the tension force applied by the string to the stopper. Thus, for any short time interval during the rotation,

$$W = (F\cos\theta)\Delta d$$
$$= F(\cos 90°)\Delta d$$
$$W = 0.0\ \text{J}$$

Adding the work done over all the short time intervals in half a revolution yields 0.0 J. Therefore, the work done by the tension on the stopper is zero.



KINETIC ENERGY

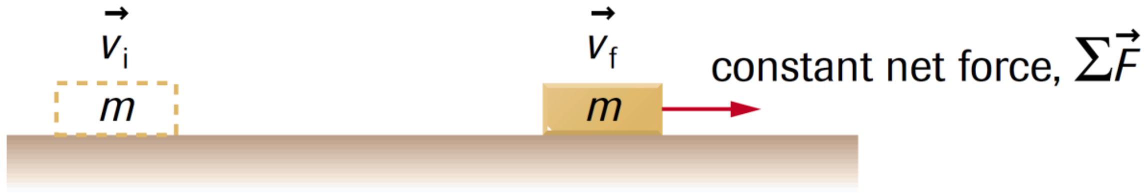
- **Kinetic Energy (E_K) [J]:** the energy of motion.

We define kinetic energy as

$$E_K = \frac{1}{2}mv^2$$

KINETIC ENERGY

- **Kinetic Energy (E_K) [J]:** the energy of motion
- To derive an equation for E_K, lets look at total work done to change the velocity of an object from $\vec{v}_i$ to $\vec{v}_f$:



$$W_{total} = (\Sigma F)(\cos\theta)\Delta d$$

with $\theta = 0° \Rightarrow \cos 0° = 1$, and $\Sigma F = ma$,

$$W_{total} = (ma)(1)\Delta d$$

- Rearranging $v_f^2 = v_i^2 + 2a\Delta d$ for a, we get

$$W_{total} = \left(m\frac{v_f^2 - v_i^2}{2\Delta d}\right)\Delta d$$

$$W_{total} = \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2$$

WORK-ENERGY THEOREM

- From before, we know $E_K = \frac{1}{2}mv^2$. This allows us to write work as

$$W_{total} = E_{Kf} - E_{Ki}$$
$$= \Delta E_K$$

- **Work-Energy Theorem:** The total work done on an object equals the change in the object's kinetic energy, provided there is no change in any other form of energy (for example, gravitational potential energy).
- This holds true in all three dimensions.

PROBLEM 4

What total work, in megajoules, is required to cause a cargo plane of mass 4.55×10^5 kg to increase its speed in level flight from 105 m/s to 185 m/s?

PROBLEM 4 – SOLUTIONS

$m = 4.55 \times 10^5$ kg

$v_i = 105$ m/s

$v_f = 185$ m/s

$W_{total} = ?$

$$W_{total} = \Delta E_K$$
$$= E_{Kf} - E_{Ki}$$
$$= \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$
$$= \frac{1}{2}m\left(v_f^2 - v_i^2\right)$$
$$= \frac{1}{2}\left(4.55 \times 10^5 \text{ kg}\right)\left((185 \text{ m/s})^2 - (105 \text{ m/s})^2\right)$$
$$= 5.28 \times 10^9 \text{ J}\left(\frac{1 \text{ MJ}}{10^6 \text{ J}}\right)$$
$$W_{total} = 5.28 \times 10^3 \text{ MJ}$$

The total work required is 5.28×10^3 MJ.

PROBLEM 5

A fire truck of mass 1.6×10^4 kg, travelling at some initial speed, has -2.9 MJ of work done on it, causing its speed to become 11 m/s. Determine the initial speed of the fire truck.

PROBLEM 5 – SOLUTIONS

$m = 1.6 \times 10^4$ kg

$\Delta E_K = -2.9$ MJ $= -2.9 \times 10^6$ J

$v_f = 11$ m/s

$v_i = ?$

$$\Delta E_K = \frac{1}{2}m(v_f^2 - v_i^2)$$

$$2\Delta E_K = mv_f^2 - mv_i^2$$

$$mv_i^2 = mv_f^2 - 2\Delta E_K$$

$$v_i^2 = \frac{mv_f^2 - 2\Delta E_K}{m}$$

$$v_i = \pm\sqrt{\frac{mv_f^2 - 2\Delta E_K}{m}}$$

$$= \pm\sqrt{\frac{(1.6 \times 10^4 \text{ kg})(11 \text{ m/s})^2 - 2(-2.9 \times 10^6 \text{ J})}{1.6 \times 10^4 \text{ kg}}}$$

$$v_i = \pm 22 \text{ m/s}$$

We choose the positive root because speed is always positive. The initial speed is thus 22 m/s.

SUMMARY – WORK DONE BY A CONSTANT FORCE

- Work is the energy transferred to an object when a force F, acting on the object, moves it through a distance d.
- The SI unit of work is the joule (J).
- If the force causing an object to undergo a displacement is at an angle to the displacement, only the component of the force in the direction of the displacement does work on the object.
- Under certain conditions, zero work is done on an object even if the object experiences an applied force or is in motion.

SUMMARY – KINETIC ENERGY & THE WORK ENERGY THEOREM

- Kinetic energy E_K is energy of motion. It is a scalar quantity, measured in joules (J).
- The work-energy theorem states that the total work done on an object equals the change in the object's kinetic energy, provided there is no change in any other form of energy.

PRACTICE

Readings

- Section 4.1, pg 178
- Section 4.2, pg 184

Questions

- pg 183 #1-3,5-7
- pg 188 #1-8